

Work $\Rightarrow$ If an external force acting on a particle having position vector $\vec{r}$. It through a displacement $d\vec{r} = d\vec{r}$.
 $dW = \vec{F} \cdot d\vec{r}$

Power $\Rightarrow$ Power of an agent is defined as the time rate of doing work.

$$P = \frac{dW}{dt}$$

$$P = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

Energy $\Rightarrow$ Energy of a particle is its capacity of doing work. It is the measure of the amount of work that a body can do.

Kinetic energy $\Rightarrow$ It is the type of energy possessed by a particle by virtue of its motion.

$$T = \frac{1}{2}mv^2$$

Conservative Force $\Rightarrow$ A Force is said to be Conservative, if the work done by it in moving a particle from one point to another in space depends only on the position. (initial to final) of these points and not on the actual path following by the particle.

$$W_{pa} = \int_p^a \vec{F} \cdot d\vec{r} = \int_p^a \vec{F} \cdot d\vec{r} = \int_p^a \vec{F} \cdot d\vec{r}$$

$$\oint \vec{F} \cdot d\vec{r} = 0$$

2. # Non-Conservative Force $\Rightarrow$ A Force is said to be non-Conservative, if the work done by it on a particle that moves between two points. Depends upon the path followed by the particle.

Conservative Theorem of Linear momentum of a particle $\Rightarrow$

If total force acting on a particle is zero, then the linear momentum ($\vec{p}$) is Constant (or Conserved).

• Acc. to Newton's second law of motion.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{F} = 0 \quad \frac{d\vec{p}}{dt} = 0$$

Integrating we get

$$\vec{p} = \text{Constant}$$

Conservation theorem for the angular momentum of particle $\Rightarrow$

If the total torque $\vec{\tau}$ acting on a particle is zero, then the angular momentum ($\vec{L}$) is Conserved.

$$\vec{L} = \vec{r} \times \vec{p}$$

The moment of the total force $\vec{F}$ or torque $\vec{\tau}$ on particle

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \vec{r} \times \frac{d(mv)}{dt}$$

Taking total time derivative

$$\frac{d}{dt}(\vec{r} \times \vec{p}) = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

$$= \vec{\omega} \times \frac{d}{dt}(m\vec{v}) + \vec{v} \times m\vec{v}$$

Now, the second term expression

$$\vec{v} \times m\vec{v} = 0$$

$$\frac{d}{dt}(\vec{\omega} \times \vec{p}) = \vec{\omega} \times \frac{d}{dt}(m\vec{v})$$

$$\tau = \frac{d}{dt}(\vec{\omega} \times \vec{p})$$

$$\tau = \frac{d}{dt}(\vec{L})$$

Thus, the time rate of change of angular momentum of a particle is equal to the vector torque acting on it.

$$\frac{d\vec{L}}{dt} = 0$$

Integrating we get

$$\vec{L} = \text{Constant.}$$

Conservation theorem of energy of a particle :->

This theorem states that if the force acting on a particle are conservative, then the total energy of a particle, which is the sum of its kinetic energy T and potential energy V is constant or conserved or $T+V = \text{Constant}$.

$$dW = \vec{F} \cdot d\vec{s}$$

The total work W_{12} done by the external force $\vec{F}$,

$$\begin{aligned} W_{12} &= \int_1^2 dW = \int_1^2 \vec{F} \cdot d\vec{s} \\ &= \int_1^2 \frac{d\vec{p}}{dt} \cdot \frac{d\vec{s}}{dt} dt \end{aligned}$$

$$\text{Moment of inertia of the shell} = \frac{2}{3} (\text{mass of the shell}) \times (\text{radius})^2 \quad 3$$

$$= \frac{2}{3} \left[\frac{3M}{R^3} x^2 dx \right] \times x^2 = \frac{2M}{R^3} x^4 dx$$

The moment of inertia of the sphere about a diameter.

$$= \frac{2M}{R^3} \int_0^R x^4 dx = \frac{2M}{R^3} \left[\frac{x^5}{5} \right]_0^R = \frac{2M}{R^3} \left[\frac{R^5}{5} \right]$$

$$= \frac{2}{5} MR^2$$